Paradoxes Explained

The Twin Paradox

Let's imagine that one twin travels to a star 6 light-years away from the earth with a constant speed of 0.5c. Once there, they turn around and return to the earth at the same speed. Let's see how much time has passed for both twins. To greatly simplify the situation, we will not worry about any accelerations. (Taking that into account would be a lot more complicated and ultimately just make the trips take a little longer and the differences in ageing less without changing the spirit of the problem.)

The twin who stayed behind (A) was always in the same inertial reference frame. According to A, twin B traveled to a place that is 6 light-years away at half the speed of light. Note that distance is a proper length, since it is at rest with twin A and the earth. So according to A we have

$$L_o = vt \quad \longrightarrow \quad 6 = (0.5)t$$

According to twin A, it took B 12 years to reach the star. The return trip would be the same math, so another 12 years. **Twin A will be 24 years older when B returns to earth**. Note that those two times were time dilated times because the starting and ending points were at different locations in their reference frame.

How long did the trips take according to B? The key is to recognize that twin B saw a length contracted distance go by them at 0.5c and that they measured the proper time for the trip (you can't move with respect to yourself.)

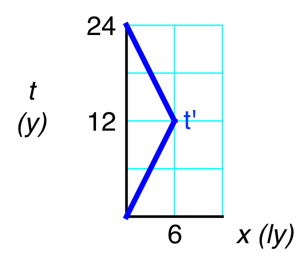
$$L = \frac{L_o}{\gamma} = 6\sqrt{1-0.5^2} = 5.196$$

Then we can calculate the time it took B to travel to the star

$$L = vt_o \quad \longrightarrow \quad 5.196 = (0.5)t_o$$

According to B it took 10.4 years for the star to reach them. The trip back would be another 10.4 years, so **Twin B will have aged 20.8 years when they return to earth.** Notice that twin B will have been in multiple reference frames.

We can show this a little easier with a spacetime diagram, shown to the right. The blue worldline is the ship moving away from the earth and then returning to the earth. Clearly from the earth frame, it will take the ship 12 years to travel 6 light years moving at half the speed of light, and then another 12 years to get back to the earth. Obviously 24 years passed on the earth.



How many years passed on the ship? Remember that the scale on the t' axis is different from the t axis. (Go back to the *Spactime Diagrams 2* handout.) The ship turns around at t = 12 years, so that corresponds to

$$t' = \frac{t}{\gamma} = t\sqrt{1-\beta^2} = 12\sqrt{1-0.5^2} = 10.4\ years$$

According to the ship, it took 10.4 years to get there, and then another 10.4 years to return, so only 20.8 years passed on the ship. The twin on the ship aged less than the twin who stayed on the earth! The differences would get bigger the faster the speeds.

Keep in mind that the twin on the earth stayed in the same inertial reference frame the entire time. The twin on the ship had to accelerate three different times, so was in multiple reference frames, which breaks the symmetry. **The twin who goes on the trip ages less than the twin who stayed behind.**

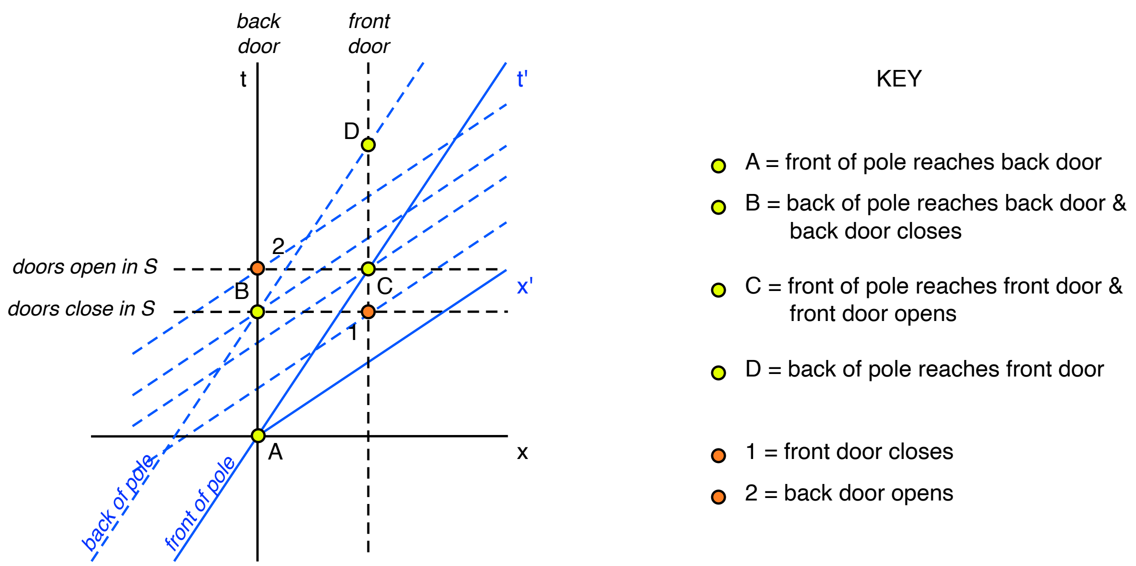
Paradoxes Explained

The Barn and Pole Paradox

The key idea is to remember that two events separated in space can only be simultaneous in one reference frame. So while the doors can close simultaneously and then open simultaneously in the barn reference frame, they will not be simultaneous according to the pole. According to the barn, the pole is completely inside it for a small amount of time, so both doors can be closed for a brief amount of time.

According to the pole, both ends of the pole are sticking out the barn for a small amount of time, and so the doors cannot both be closed at the same time. According to the pole, the barn is moving past it, and if the barn says the doors closed simultaneously, the pole will say the doors did not close simultaneously. According to the pole, after it has just reached the barn, the far door will close and then open just before it reaches the front of the pole. Then, after the rear of the pole as reached the barn, that barn door will close briefly, and then actually open again before the pole leaves the barn.

This is a lot clearer with a spacetime diagram. In the following diagram, the barn will be the S frame. We will call the front of the pole the S' frame, and also show the back of the pole as another worldline. The pole will be moving to the right according to the barn. For convenience, the origins of both frames are when the front of the pole is at the back door.

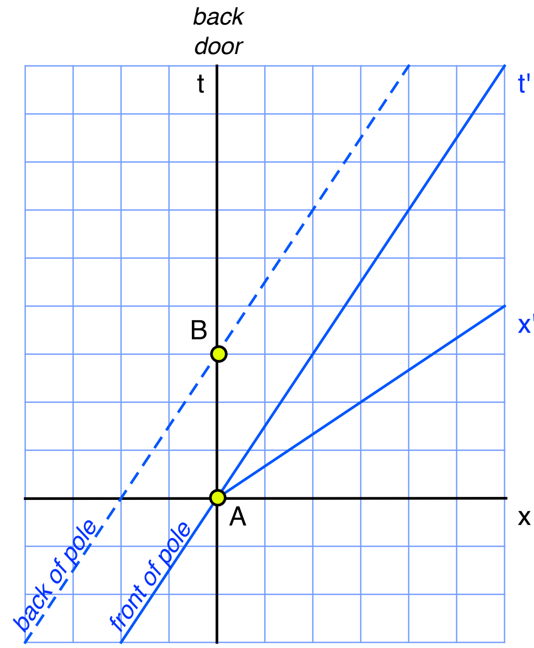


The 4 parallel dashed lines are the lines of simultaneity for S'. There is a brief time in S in which the doors are both closed and the pole is inside it. However, in S' the front door closes and then opens before the back of the pole even reaches the back door (and before the front of the pole reaches the front door, and the front of the pole has passed the front door before the back of the pole reaches the back door.

Even though I said it was "a lot easier with a spacetime diagram,' jumping into that diagram all at once can be daunting. Let's step through the spacetime diagram for the barn-pole paradox a piece at a time.

Paradoxes Explained

First, consider the diagram shown to the right. It shows three worldlines: the back door of the barn, the back of the pole and the front of the pole. In addition, there are two labeled events, A and B.



1. Which is at rest, the barn or the pole and how do you know?

2. What is happenging at event A?

3. What is happenging at event B?

4. How fast is the pole moving?

5. From the diagram, you can determine the length of the pole according to the barn. What is that length?

6. Calculate the proper length of the pole.

7. To make a barn the same proper length as the pole, draw in the correct worldline of the front door of the barn in the diagram above.

8. Draw and label event C which will be the front of the pole reaching the front door.

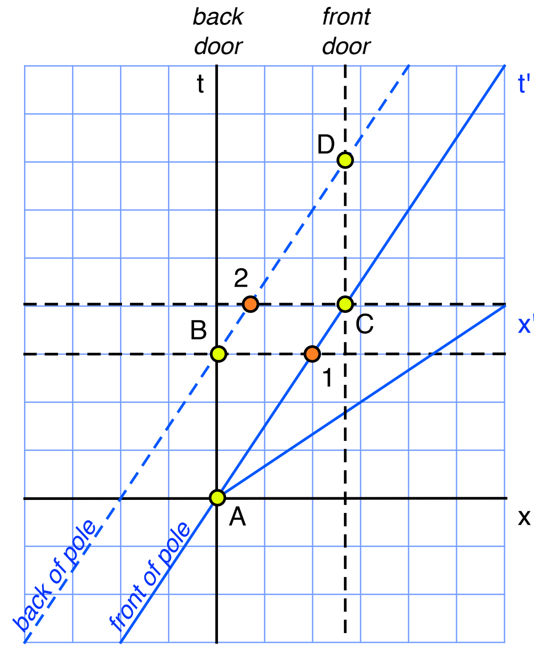
9. Draw and label event D which will be the back of the pole reaching the front door.

10. Draw and label event 1, which will be the front door closing simultaneous with the back of the pole reaching the back door, which is when the back door will close.

11. Draw and label event 2, which will be the back door opening simultaneous with the front of the pole reaching the front door, which is when the front door will open.

Paradoxes Explained

Hopefully, your diagram from the other side looks like the one to the right. (Notice that both events B and C are actually two things happening at the same place and same time.)



In the S frame it is easy to see that the doors could potentially close and open simultaneously, thus "trapping" the pole inside the barn. You drew it that way. To see what happens in the S' frame, we need to see where those events are on the t' axis.

12. Draw in 4 lines parallel to the x' axis, with 1 line going through each of the events B, C, 1 and 2. (*You should now have the spacetime diagram shown on page 2.*)

13. What do the dashed lines you just drew represent?

13. Starting with A, describe what happens according to the pole.